## Exercises- Predicate Logic

**Exercise 1**

Transform the following sentences from natural language into predicate formulas.

Explain the syntactic elements used in the predicate formulas: variables, constants,

functions symbols, predicate symbols.

2. In a plane there are lines parallel to a constant line and there are lines perpendicular to .



7. For every positive integer , if is not a prime, then there exists a prime such that



divides and is less than



V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

**Exercise 2**

Transform the following statements from natural language into predicate formulas choosing

the appropriate constants, function symbols and predicate symbols:

5. Anyone who owns a rabbit hates anything that chases any rabbit.

12. Every animal either likes to eat all plants or all animals much smaller than itself that

like to eat some plants.

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

**Exercise 3**

Using the given interpretation evaluate the following formula:

U7= ((ꓯx)A(x) ->(ꓱx)B(x))-> ((ꓱx)A(x)->(ꓯx)B(x))

Interpretation , where:



the set of all persons from Romania



”the person lives in a city”;



”the person has a job”.



V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

vI(U7)= vI(((ꓯx)A(x) ->(ꓱx)B(x)) -> ((ꓱx)A(x)->(ꓯx)B(x)))

1. = vI((ꓯx)A(x) ->(ꓱx)B(x)) -> vI((ꓱx)A(x)->(ꓯx)B(x))
2. =( vI((ꓯx)A(x)) -> vI((ꓱx)B(x)) ) -> (vI(((ꓱx)A(x)) -> vI((ꓯx)B(x)))
3. =( (ꓯx in D)(”x lives in city”) -> (ꓱx in D)(“x has a job”)) ->
4. ((ꓱx in D)(“x lives in a city”) -> (ꓯx in D)(“x has a job”))
5. = (F ->T) ->(T ->F)
6. =T ->F
7. =F
8. I is an anti-model of U7

Interpretation I1 = <D,m> , where:

the set of all persons from Romania

m(A)(x): ”the person x is alive”;

m(B)(x): ”the person x is a mermaid”.

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

vI1(U7)= vI1(((ꓯx)A(x) ->(ꓱx)B(x)) -> ((ꓱx)A(x)->(ꓯx)B(x)))

1. = vI1((ꓯx)A(x) ->(ꓱx)B(x)) -> vI1((ꓱx)A(x)->(ꓯx)B(x))
2. =( vI1((ꓯx)A(x)) -> vI1((ꓱx)B(x)) ) -> (vI1(((ꓱx)A(x)) -> vI1((ꓯx)B(x)))
3. =( (ꓯx in D)(”x is alive”) -> (ꓱx in D)(“x is a mermaid”)) ->
4. ((ꓱx in D)(“x is alive”) -> (ꓯx in D)(“x is a mermaid”))
5. = (T ->F) ->(T ->F)
6. =F ->F
7. =T
8. I1 is an model of U7

**Exercise 4**

Choose an arbitrary interpretation with a finite domain for the formula U7 and prove that it is a model of U7.

U7= ((ꓯx)(A(x) ->B(x))-> ((ꓱx)A(x)->( ꓱx)B(x))

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

vI(U7)=

(ꓯx in {1,3})P(x) **≡P(1) ^P(3)**

(ꓱ x in {1,3})P(x) **≡P(1) vP(3)**

Let us consider the interpretation I= <D, m>, where:

•D = {6,8} , the domain of interpretation

•m(A):{6,8}→{T, F}, m(A)(x)="x⋮4"

m(B):{6,8}→{T, F}, m(B)(x)="x⋮2"

1. vI (U7)= vI (((ꓯx)(A(x) ->B(x))-> ((ꓱx)A(x)->( ꓱx)B(x)))

= vI (((ꓯx)(A(x) ->B(x)))-> vI ((ꓱx)A(x)->( ꓱx)B(x)))

= vI (((ꓯx)(A(x) ->B(x))-> (vI (ꓱx)A(x)-> vI ( ꓱx)B(x)))

=((6⋮4 -> 6⋮2)^(8⋮4->8⋮2))->( (6⋮4)V(8⋮4) -> (6⋮2)V(8⋮2))

=(F->T)^(T->T)->((FVT)->(TVT))

=T^T->(T->T) =T->T =T

I is a model of U7 , so U7 is consistent

**Exercise 5. Succession to the British throne**

|  |  |
| --- | --- |
| *Hypotheses*:  . If is the king and is his oldest son, then  can become the king.  . If is the king and defeats ,  then will become the king.  . is the king.  . defeated .  . is ’s oldest son.  *Conclusion*:  . Can become the king? | variables:  constants:  predicate symbols: **unary**: ,  **binary**: , s |

Check whether the conclusion is derivable from the set of hypotheses {}



using the definition of deduction and the appropriate inference rules.

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

**Exercise 8**

Prove that the following formulas are not valid by finding anti-models for them.

1. ;



1. ;



1. ;



1. ;



V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

vI(U)=